Enhancement of Fractions from Playing a Game

Ya Ling Lee
University of Otago
<yaling@maths.otago.ac.nz>

The effectiveness of educational mathematics games were investigated by testing a game of fractions with Year 8 students. The influences of the game on students' achievement in fractions were assessed using pre-post quizzes and maths tasks. This paper focused on the achievement differences of students between pre and post quizzes. The objectives were to identify the increases of achievement from pre quiz to post quiz and the relevancy with the game. The results showed that the game enhanced students' understanding on representations of fractions.

Introduction

Educational Mathematics Games

Games are recognized as one of the useful activities for teaching mathematics (Lovitt & Clarke, 1988) and have been used in the Numeracy Development Project (NDP), a teaching mathematics professional development programme in New Zealand. Games help to improve children's knowledge, and make mathematics interesting and easier to learn (Fisher & Neill, 2007; Young-Loveridge, 2005). Games motivate students and promote mathematics learning (Ke & Grabowski, 2007; Gueron, 2001; Koirala & Goodwin, 2000; Peters, 1998). Nevertheless, learning outcomes related to the games should be clearly specified to make the usefulness of games explicit to students (Bragg, 2007). Teachers are doubtful of and need assurance in the learning objectives achieved by students from playing games by themselves (Ell, 2007).

The Study

A study has been conducted to investigate the use of games in learning mathematics by testing a game of fractions with Year 8 students. The game of fractions integrates the mathematical content of comparing sizes of fractions with the game idea of forming staircases of "fraction bricks". The learning objectives of the game are to order fractions from smallest to largest and vice versa. Order and equivalence are the most basic yet critical topics in fractions (Smith, 1995; Behr, Wachsmuth, Post & Lesh, 1984; Streefland, 1993). More effort on fractions and proportional reasoning is needed in the NDP (Holton, 2007) due to the unsatisfactory performance of students in fractions (Young-Loveridge, 2006; Ward & Thomas, 2007). In this study several assessment tools were used to identify students' feedback of the game and students' achievement in fractions. Mathematical learning should be emphasized than motivation of students in educational game software (Kafai, Franke & Battey, 2002). Hence, the enhancement of fractions influenced by the game was specifically assessed using pre-post quizzes and pre-post maths tasks. This paper focuses on the achievement differences of students between the pre and post quizzes. The objectives of this paper were to identify the increases of achievement from pre quiz to post quiz and the relevance of the game in any changes. Suggestions are also made in this paper for the learning of other aspects of fractions that are not yet present in the game.

Theoretical Framework of the Game

A basic maths game model was developed to integrate instructional factors of mathematics games (Booker, 2000, 2004) and key structural elements of games (Prensky, 2001). Several factors determine the effectiveness of the game in mathematics learning such as conceptual analysis of fractions, students' misconceptions, game structure, instructional strategies, student methods and thinking, and relationship of the game to the New Zealand Number Framework. Six key elements of designing a game structure are story, rules, goals and objectives, outcomes and feedback, conflict/challenge and interaction. The model was improved by incorporating feedback from students with different mathematical abilities (Lee, 2007). To ensure the mathematical usefulness of the game, the criteria of educational software (on rational numbers) reviews (Kafai et al., 2002) was referred to, which includes categories of mathematical topic, students' strategies, representation, context and integration.

Digital Outputs of the Game

The game world occurs in a tower that a boy who is lost in the woods wants to climb to see the way home. Rectangular bricks that represent sizes of fractions and are labelled with symbols of fractions are displayed at the side for the boy to use to form staircases. The rules of the game are to order tall (vertical) fraction bricks (Figure 1) from smallest to largest and long (horizontal) fraction bricks from largest to smallest (Figure 2). While moving up, the boy has to jump up or duck down to avoid creatures to earn points for the game. When the bricks are ordered incorrectly, specific feedback and tips are given to guide students so that they can continue the game. Specific feedback is needed to remind players of incorrect orderings caused by the belief that the numerical value of a fraction is represented by two independent natural numbers (Stafylidou & Vosniadou, 2004). The tips pages include various strategies that can be used to order fractions such as finding a common denominator, numerical conversions and using reference points.

Figure 1: Tall bricks

Figure 2: Long bricks

Figure 3: Broken bricks

Pedagogical Approaches of the Game

Three representations of bricks were designed for the game context of fraction brick staircases to achieve different pedagogical goals. **Visible bricks** (Figure 1) represent sizes of fractions that can be seen and so enable students to connect symbols and representations of fractions, and compare fractions in a concrete way. **Broken bricks** (Figure 3) are divided in parts that are manipulable and so allow students to select parts from the whole and see the consequent changes of symbols of fractions, and most importantly compare fractions with unlike denominators in the same whole. **Hidden bricks** with only symbols of fractions labelled on them require students to interpret fraction symbols and judge the sizes of fractions independently from the mathematical notation. Moving from visible, broken to hidden bricks as the game progresses, students also experience fractions from concrete representations to abstract symbols. The increasingly challenging environment promotes the comprehension of students of the relations between manipulative and abstract mathematical symbols (Uttal, Scudder & DeLoache, 1997).

Method

Sample

Approximately 150 Year 8 students (11 to 13 years old) from three intermediate schools in a small city participated in the study. A class of twenty seven students was selected for a preliminary investigation and these results are discussed in this paper. The class consisted of fifteen male and twelve female students. The students have learned fractions as a key aspect in the number strand of mathematics in the New Zealand curriculum. They had explored halves, quarters, thirds and fifths; found fractions of whole number and decimal amounts; named equivalent fractions; and converted between fractions, decimals and percentages. Therefore, the students all had some knowledge of fractions before taking part in the research.

The Design

Every participant took about an hour of school time to play the digital game of fractions and complete several assessment tools such as pre and post maths tasks, pre and post online quizzes, and online questionnaires. The pre and post quizzes were multiple choice questions of fractions of similar difficulty. Pre and post maths tasks asked students to order fractions from smallest to largest and explain their methods. The questionnaires were used to obtain students' feedback about the game and learning fractions using the game. The game recorded the game play data of every student, including the number of attempts to order fractions correctly, the number of additional questions chosen, and the frequency of accessing the tips pages of the game. Note that no formal classroom teaching took place during this hour of the students' time.

Results and Discussion

Twenty four students completed pre and post quizzes and their achievement were analysed to determine the achievement differences: wrong in the pre quiz but right in the post quiz (WR), right in the pre quiz but wrong in the post quiz (RW), wrong in the pre and post quizzes (WW) and right in the pre and post quizzes (RR) (Table 1).

Table 1
Achievement Differences Between Pre and Post Quizzes

Questions	Number of students according to their responses on the pre and post quizzes for each question			
	WR	RW	WW	RR
1	3	0	0	21
2	2	1	1	20
3	3	2	2	17
4	6	6	1	11
5	3	5	7	9
6	1	0	17	6

The results showed that students achieved better in the first three questions about representations of fractions, than the last three questions about ordering and operating with fractions. In the first three questions, eight students increased (WR), one to two students decreased (RW) and were wrong twice (WW) in the first three questions. On the other hand, ten students increased (WR), but none to six students decreased (RW) and one to seventeen students were wrong twice (WW) in the last three questions. Among the questions focusing on representations of fractions, question 1 seemed to be more relevant to the game because all students were right in the post task after playing the game. Meanwhile, question 6 was the least influenced by the game because only one student increased (WR) after the game but seventeen students were wrong in both pre and post quizzes (WW). Few students improved from the pre to the post quiz as the Year 8 students were supposed to know about fractions prior to playing the game. The impacts of the game were noticed from the increases (WR) and decreases (RW) resulting from enhancement of or challenging towards the unstable knowledge of fractions. The increase of the achievement from wrong in the pre quiz to right in the post quiz (WR) is particularly focused on in the following discussions to highlight the positive influence of the game.

Representations of Fractions

Fractions were presented as shaded parts in different rectangular divided quantity diagrams in questions 1 to 3.

Rectangular divided quantity diagram. In question 1, one student thought $\frac{3}{5}$ was made up of five shaded and three unshaded parts (C in Figure 4) while another student thought $\frac{2}{5}$ was made up of two shaded and five unshaded parts (D in Figure 5). In question 2, two students estimated the rectangles with five out of eight parts shaded (Figure 6 and 7) as "$\frac{1}{2}$". The broken bricks (Figure 3) were used in the game presented fractions in divided parts and the shaded parts (in blue) of broken bricks made the representations of fractions explicit to students. The manipulation of broken bricks engaged students with representations of fractions, a key to building conceptual knowledge in fractions (Kafai et al., 2002). By splitting parts from a whole, students were led to understand fractions as being related of numerators and denominators rather than to two independent natural numbers (Stafylidou & Vosniadou, 2004). For example, when a half part was separated from a whole, the symbol changed from $\frac{2}{2}$ to $\frac{1}{2}$. Hence, after playing the game, in the post quiz, students were able to recognize $\frac{3}{5}$ as a fraction with three out of five parts shaded, $\frac{2}{5}$ as a fraction with two out of five parts shaded, and the rectangles with five out of eight parts shaded to represent an exact fraction of "$\frac{5}{8}$", respectively.

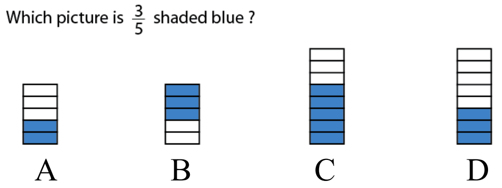

Figure 4

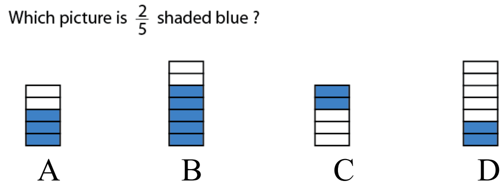

Figure 5

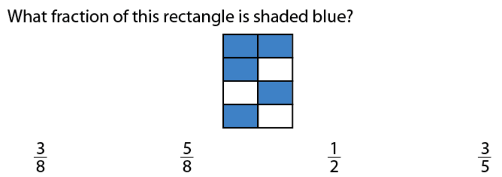

Figure 6

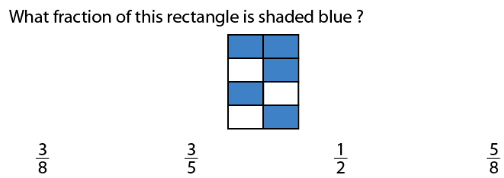

Figure 7

Different types of divided quantity rectangles. All students were able to determine fractions represented in rectangular divided quantity diagrams like the broken bricks used in the game. Two students were still confused with other types of rectangular divided quantity diagrams (Figure 6 or 7) that they were unfamiliar with. One of them who had first thought "$\frac{3}{5}$" was represented by the rectangle with three parts unshaded and five parts shaded (Figure 6) changed to "$\frac{3}{8}$" (Figure 7) after playing the game. Another student who had chosen "$\frac{5}{8}$", which was right to represent fractions of five out of eight parts shaded (Figure 6) in the pre quiz, however, chose "$\frac{3}{8}$" (Figure 7) in the post quiz. The game could be changed to include different types of representations of fractions to encourage students to imagine fractions more flexibly.

Continuous representations of fractions. Three students had improved but four others were still struggling with the range of fractions (Figures 8 and 9) in question 3. In fact, the question was related to the conceptual knowledge of fractions as parts of a whole, as had been emphasized with the broken bricks played in the game. However, the effect of the game was limited here because the learning outcomes of the game were only indirectly related to the continuous representations of fractions.

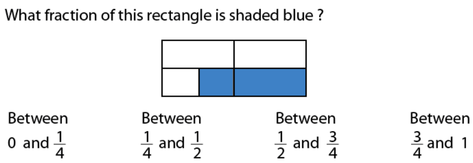

Figure 8

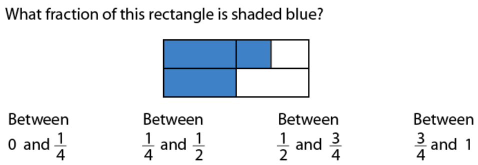

Figure 9

Ordering and Operating with Fractions

Labelling fraction bricks with mathematical notation enables students to connect representations and symbols of fractions but is not sufficient for them to engage with ordering of fractions. However, some influence of the game on the knowledge of ordering and operating with fractions was still found in students' achievements.

Refer fractions to a half. Comparing fractions $\frac{5}{9}$ and $\frac{6}{13}$ in visible and hidden bricks gave students a mental picture of fractions that were close to a half. They could imagine that if half of 9 was 4.5, then 5 out of 9 parts or $\frac{5}{9}$ was larger than a half. The method of referring to a half was simpler than numerical conversions and finding a common denominator when comparing fractions close to a half like $\frac{5}{9}$ and $\frac{6}{13}$. The mental picture of fractions also reminded students of the misconception of big numbers equal to big fractions, in which fractions that consist of big numbers (i.e., $\frac{6}{13}$) are bigger than fractions that consist of small numbers (i.e., $\frac{5}{9}$). When asked "Which is smaller or larger?" in question 4, six students who had chosen $\frac{5}{9}$ was smaller or $\frac{6}{13}$ was larger in the pre quiz chose $\frac{5}{9}$ was larger or $\frac{6}{13}$ was smaller in the post quiz. On the other hand, the other six students who had been right in the pre quiz were wrong in the post quiz. Two students chose $\frac{5}{9}$ was larger in the pre quiz but smaller in the post quiz and another student chose $\frac{6}{13}$ was smaller in the pre quiz but larger in the post quiz. In the post quiz, two students thought $\frac{5}{9}$ and $\frac{6}{13}$ were the "same" and another student thought it was "impossible to tell" which fraction was larger. Obviously the inconsistent performances between pre and post quizzes were due to an unstable knowledge of fractions, which could probably be strengthened by practicing more similar fractions that were close to a half.

Strategies for ordering fractions. In question 5, two students who had chosen "$\frac{2}{9},\frac{3}{18},\frac{4}{6},1$" for the order from smallest to largest realized afterwards that $\frac{3}{18}$ was actually smaller than $\frac{2}{9}$ or $\frac{4}{18}$, an equivalent fraction in a common denominator of 18. In fact, finding common denominators was one of the strategies highlighted in the tips pages of the game. More activities could be included in the game for students to find equivalent fractions in common denominators because many students were still lacking in comparing fractions with unlike denominators $\frac{2}{9}$, $\frac{4}{6}$ and $\frac{3}{18}$. Three students who had been right in the pre quiz chose "$\frac{2}{9}, \frac{3}{18}, \frac{4}{6}, 1$" as from smallest to largest and "$1, \frac{2}{9}, \frac{3}{18}, \frac{4}{6}$" as from largest to smallest respectively in the post quiz. The strategy of finding a common denominator could be enhanced if students were asked to find the facts that $\frac{2}{9} = \frac{4}{18}$ and $\frac{4}{6} = \frac{12}{18}$ to compare them with $\frac{3}{18}$.

Addition of fractions. Only one student who had chosen "17" as the closest to $\frac{6}{7} + \frac{9}{10}$ in the pre task chose "2" as the closest to $\frac{10}{11} + \frac{6}{7}$ in the post task. More than half of the students were wrong in the pre and post tasks because they added up numerators or denominators of fractions. The concept of fractions as parts of a whole had been emphasized in the game to overcome the belief of fractions as two independent numbers. The parts of $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{6}{7}$ could be selected and split from the whole of $\frac{3}{3}$, $\frac{5}{5}$ and $\frac{7}{7}$ in broken bricks. Unfortunately, this did not appear to be helpful in estimating that $\frac{6}{7}$, $\frac{9}{10}$ and $\frac{10}{11}$ were close to one in order to get an approximate idea of their sum in question 6.

Conclusions

The achievement differences between the pre quiz and the post quiz has suggested that the impact of the game is minimal, particularly on the representations of fractions. After manipulating parts of fractions on broken bricks in the game, all students were able to determine fractions of the shaded parts of rectangular divided quantity diagrams in the post quiz. However, some students were still confused with other types of representations of fractions that were not played in the game. Generally, students achieved better in the questions related to representations of fractions than the questions related to ordering and operating with fractions. While some students improved in comparing fractions that were close to a half and with unlike denominators, others were did not improve and some did worse from the pre quiz to the post quiz. Students relied on visualizing the sizes of bricks then interpreting the symbols of fractions when playing the game. No significant effect of the game was detected when estimating fractions close to one but this was not stressed in the game.

It would seem that the game that was designed for comparing sizes of fractions has particularly enhanced students' understanding of representations of fractions in rectangular divided quantity diagrams. This is attributed to the staircases of fraction bricks that made the representations of fractions meaningfully and directly applicable. The learning outcome should be specific especially when the game was played as a one-off activity in the classroom. The game could be further expanded to provide in-depth learning that covers various types of representations of fractions, more fractions close to a half and one, and strategies of ordering fractions.

References

Behr, M. J., Wachsmuth, I., Post, T. R., & Lesh, R. (1984). Order and equivalence of rational numbers: A clinical teaching experiment. *Journal for Research in Mathematics Education,* 15, 323-341.

Booker, G. (2004). Playing to win: using games for motivation and the development of mathematical thinking. *Proceedings of the International Conference: The Future of Mathematics Education, Poland.* Retrieved 20 July 2007 from the World Wide Web: http:// math.unipa.it/~grim/21_project/CiechBooker.pdf

Booker, G. (2000). *The maths game: Using instructional games to teach mathematics.* Wellington: NZCER.

Bragg, L. (2007). Students' conflicting attitudes towards games as a vehicle for learning mathematics: A methodological dilemma. *Mathematics Education Research Journal,* 19(1), 29-44.

Ell, F. (2007). Keeping going at country school: Sustaining numeracy project practices. In *Findings from the New Zealand Numeracy Development Project 2006.* Wellington: Ministry of Education.

Fisher, J., & Neill, A. (2007). Exploratory study of home-school partnership: Numeracy. In *Findings from the New Zealand Numeracy Development Project 2006.* Wellington: Ministry of Education.

Gueron, S. (2001). A game-like activity for learning Cantor's theorem. *The College Mathematics Journal,* 32, 122-125.

Holton, D., (2007). Foreword. In *Findings from the New Zealand Numeracy Development Projects 2006*. Wellington: Ministry of Education.

Kafai, Y. B., Franke, M. L., & Battey, D. S. (2002). Educational software reviews under investigation. *Education, Communication & Information,* 2(2/3), 163-180.

Koirala, H. P., & Goodwin, P. M., (2000). Teaching algebra in the middle grades using math magic. *Mathematics Teaching in the Middle School,* 5, 562-565.

Ke, F., & Grabowski, B. (2007). Gameplaying for maths learning: Cooperative or not? *British Journal of Educational Technology,* 38(2), 249-259.

Lee, Y. L. (2007). A basic maths game model for learning fractions. *The International Journal of Learning,* 14(12), 225-234.

Lovitt, C. & Clarke, D. (1988). *The Mathematics Curriculum Teaching Programme.* Victoria: Curriculum Corporation.

Peters, S. (1998). Playing games and learning mathematics: The results of two intervention studies. *International Journal of Early Years Education,* 6(1), 49-58.

Prensky, M. (2001). *Digital game-based learning*. New York: McGraw-Hill.

Smith, III, J. P. (1995). Competent reasoning with rational numbers. *Cognition and Instruction,* 13, 3-50.

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning & Instruction,* 14(5), 503-518.

Streefland, L. (1993). Fractions: A realistic approach. In T. P. Carpenter, E, Fennema & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 289-326). Hillsdale, NJ: Lawrence Erlbaum Associates.

Uttal, D. H., Scudder, K. V., DeLoache, J. S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology,* 18, 37-54.

Ward, J. & Thomas, G. (2007). What do teachers know about fractions? In *Findings from the New Zealand Numeracy Development Project 2006.* Wellington: Ministry of Education.

Young-Loveridge, J. (2006). Patterns of performance and progress on the Numeracy Development Project: Looking back from 2005. In *Findings from the New Zealand Numeracy Development Project 2005.* Wellington: Learning Media.

Young-Loveridge, J. (2005). Students' views about mathematics learning: A case study of one school involved in the great expectations project. In *Findings from the New Zealand Numeracy Development Project 2004.* Wellington: Ministry of Education.